A Simulation of Gravitational Lensing Using Classical Optics

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Abstract

Under certain conditions, matter can bend light from an object to create multiple images of that object. This phenomenon, called gravitational lensing, also affects gravity waves. This experiment used a classical glass lens to simulate gravitational lensing. The effect of the density of the lensing matter and deviations from the inverse square law for radiation are explored. Applications include detecting dark matter and celestial events and approximating the age and size of the universe.

1 Purpose of Research

The bending of radiation paths by gravity, called gravitational lensing, has several cosmological applications, including the discovery of dark matter and the approximation of the age and size of the universe. To apply gravitational lensing to these problems, one must be able to interpret the nature of the lensed images. To do this, it is necessary to know information about the mass distribution and the distance from the observer of the lensing object.

This research uses a classical lens to simulate the effect of gravitational lensing. In doing so, we can find information about the effect of an object's density and distance from the observer on the observed light intensity.

In addition, this research studies how gravitational lensing changes the inverse square law for the decay of light intensity and gravity. Furthermore, by making an analogy between the behavior of high-frequency light and gravity waves, we can study how lensing of high-frequency gravitational waves should affect the gravity acting on a body at any given point. These results have implications for currently planned attempts to detect gravitational radiation.

2 Pertinent Scientific Literature

2.1 Gravitational Lensing

Sartori (1996) states that Einstein's theory of general relativity is based on the following two postulates: (1) Gravitational matter curves space-time, and (2) Freely falling bodies follow geodesics in space-time. These postulates predict that gravitational matter such as planets, stars, and galaxies bend the trajectory of freely falling bodies, including particles such as photons and gravitons. This effect leads to the phenomenon of gravitational lensing. Consider a star, an observer, and a gravitational body between the two, as shown in figure 1.

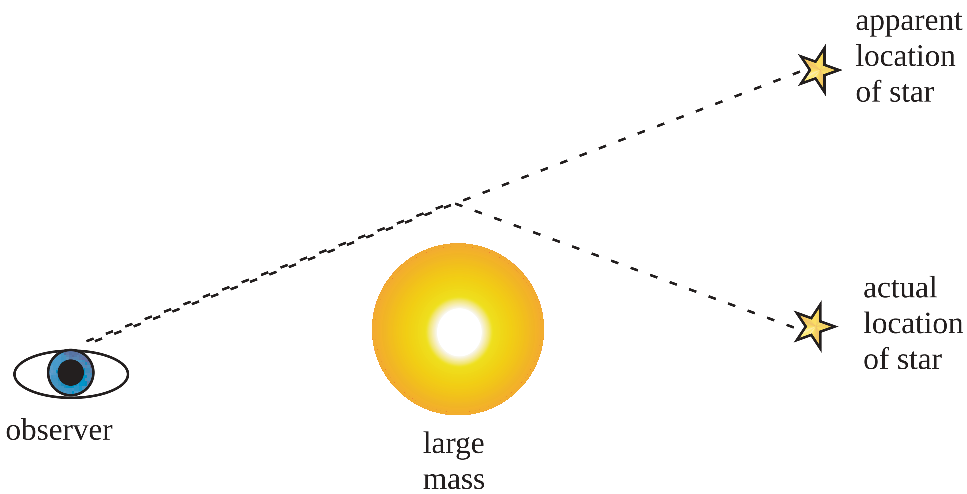

Figure 1: The Effect of Gravitational Lensing

While the star is actually behind the large mass, the light from the star is bent around the gravitational body to the observer. The observer, tracing the light ray in a straight line, perceives the star to be at a location which differs from its actual physical location. Under certain conditions of the star, mass, and observer, this phenomena can result in multiple images of the star or even a continuous ring of light around the gravitational body. As stated in Newbury (1997), the amount of light deflection caused by gravitational bodies having as large a mass as clusters of galaxies is only about 5x10^{-5} radians. Therefore, gravitational lensing only occurs with highly specific configurations of the star, the gravitational body, and the observer. However, given the vast number of galaxies and stars, numerous examples of gravitational lensing have been observed. In addition, numerous examples of multiple images have been observed, the first of which was a multiply imaged quasar observed by Walsh, Carswell, and Weyman (1979).

Gravitational lensing has several applications to cosmology. For example, Newbury (1997) explains that an important application of gravitational lensing is in investigating the mass distribution of celestial matter such as galaxies or dark matter. By studying the deflection or distortion of light made by an object, it is possible to find mass information about that object. In addition, sometimes multiple or distorted images are detected when there is no visible gravitational body that could fully account for the effect. In this way, gravitational lensing can be used to detect dark matter.

The effect of gravitational lensing can also be used in finding the Hubble Constant, H_0, which is related to the age and size of the universe. If the time delay for two images in a group of multiple images and the mass distribution of the deflector are known, the Hubble Constant can be bounded. As discussed in Koopsman et al. (in print), the Cosmic Group All-Sky Survey collaboration is one of the groups that is currently observing gravitational lensing systems to try to measure this constant.

So far, most gravitational lensing research has been performed using lensing of light radiation. However, according to the laws of general relativity both light and gravitational waves follow space-time geodesics. Therefore, observation of lensing of high-frequency gravitational waves would give the same sort of information about mass distribution as the lensing of light. New developments in gravitational radiation research are being made that will make observation of the lensing of gravity possible. Irion (2002) reports on the Laser Interferometer Gravitational-Wave Observatory (LIGO), which will be able to collect data on the intensity of high frequency gravitational waves. In addition, new sources of high frequency gravitational waves, created by rapidly spinning bodies and by sudden astrophysical events, are being discovered. For example, Sipior and Sigurdsson

(2002) discuss the possibility of double neutron star, neutron star - black hole, and black hole - black hole binary systems as sources of such radiation.

2.2 The Analogy between Classical and Gravitational Lensing

Eddington (1920) discusses several ways in which classical and gravitational lensing are analogous. The speed of light travelling through glass is lower than its speed in air or in a vacuum; similarly, the apparent speed of radiation travelling through a gravitational field is lower than its speed in the absence of gravity. In fact, we can imagine that the region around a gravitational body is filled with a medium that simulates the correct speed of light at each point. In this way, classical optics can provide an analogy for the effects of gravitational lensing. Because both light and gravitational waves follow space-time geodesics, we can use classical optics to simulate gravitational lensing of both light and gravity. Furthermore, high frequency light and gravitational waves are absorbed by bodies like stars and planets. This effect can be simulated by covering part of the optical lens with a material that absorbs light.

Lensing of low frequency gravitational radiation, for which the wavelength is comparable to the size of the lensing body, can only be treated through detailed consideration of Einstein's field equations. Therefore the results of this study will apply only to high frequency gravitational radiation.

2.3 A Note on the Inverse Square Law

As noted in Sciama (1969), in Newtonian physics, gravitational force on an object varies inversely with the square of the distance from the source. This is only an approximation to how a gravitational force behaves in general relativity; however, it is quite a good approximation. In fact, according to Eddington (1920), in many cases the Newtonian approximation is so close that including relativistic effects would only change the exponent in this law from 2 to 2.00000016. Similarly, as noted in Jenkins and White (1957), the intensity of light is inversely proportional to the s/gete of the distance from the source in Newtonian physics. In this case as well the inverse square law is a good approximation to the behavior of light in general relativity.

3 Materials and Methods

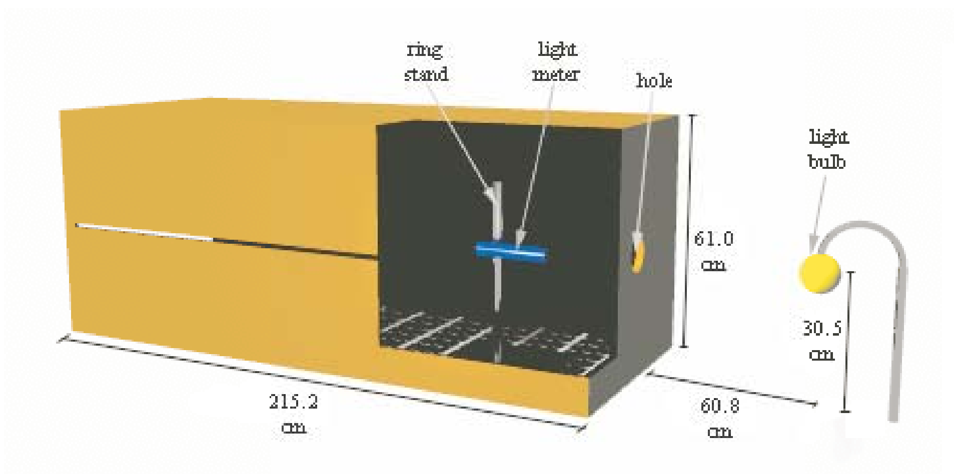

Figure 2: Cut-Away View of the Configuration of the Experiment

The experiment was designed to isolate the effect of the lens on the intensity distribution of the light radiating from a single source, so a box was built to block all other light and the experiment was conducted in a room with no extraneous light sources. The apparatus was built from two cardboard boxes measuring 61.0 cm on each side. On one of the boxes, the flaps on one end were taped closed and the flaps on the other end were taped together so that each flap was in the same plane as the side of the box to which it was attached. On the other box, the flaps on each end were taped open so that each flap was in the same plane as the side of the box to which it was attached. Then the two boxes were taped end-to-end to make a box measuring 215 by 61.0 cm by 61.0 cm, closed on one end and open on the other.

The inside walls of the box were spray painted black to minimize reflection of light. A circular hole of diameter 7 cm was centered in the closed end of the box. A lining of soft foam was taped to the edges of this hole with black duct tape to hold in the lens. A 4 cm by 4 cm grid made of twine was laid out on the floor and attached with thumbtacks to aid in positioning the light meter. A slit was cut lengthwise in one of the walls at a height of 30.5 cm above the floor to allow the cord of the light meter to attach to a computer outside the box.

As infrared light is appreciably absorbed by glass and the atmosphere, a light bulb that emitted a large amount of infrared light would make the light intensities at various positions less comparable to those measured in a vacuum. Therefore, a compact fluorescent light bulb, which emits little infrared light, was used. The 840 lumen light bulb was positioned at a horizontal distance of 60.8 cm from the lens and at a height of 30.5 cm above the ground.

The light meter was clamped to a ring stand at a height of 30.5 cm and placed inside the box. The light meter's cord was passed through the slit in the wall of the box and connected to a computer through a Vernier LabPro interface. This allowed data to be collected with the Vernier LoggerPro application and to be stored on the computer. The light meter's sensitivity range was set to be 0-600 luxes.

To streamline data collection, the LoggerPro application was set to collect light intensity data every fifteen seconds. A metronome set to click once per second and synchronized with the data collection allowed the experimenter to move the light meter along the grid while inside the box and to synchronize the movement of the light meter with the data collection.

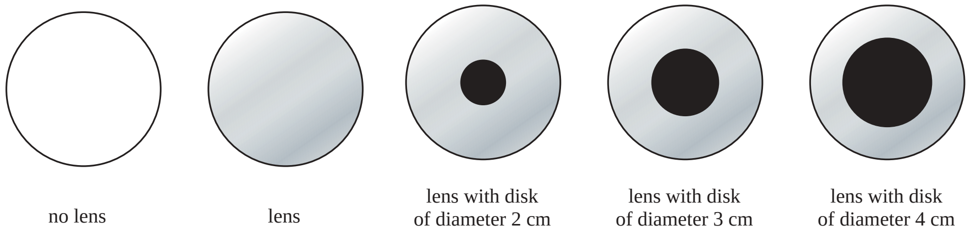

Figure 3: The Five Lens Conditions Used in the Experiment

Data were collected for five lens conditions: for no lens, for a lens, and for three sizes of opaque disks covering the center of the lens. The convex, circular lens measured 7.0 cm in diameter and 1.50 cm in thickness. The disks were cut from plastic and painted with black spray paint. The three disks measured respectively 2.0 cm, 3.0 cm, and 4.0 cm in diameter (see figure 3).

For each of these five conditions, the light intensity was measured in increments of 4.0 cm from a distance of 4.0 cm from the wall of the box containing the hole through about 140 cm from the wall. For each lens condition, light intensity data was collected along three lines: the lens axis, the line at a horizontal distance of 4 cm from the lens axis, and the line at a horizontal distance of 8 cm from the lens axis.

4 Results

4.1 No Lens (Control)

Along the lengthwise axis of the hole, the light intensity should vary approximately inversely with the square of the distance from the hole. A curve fit computed (by least

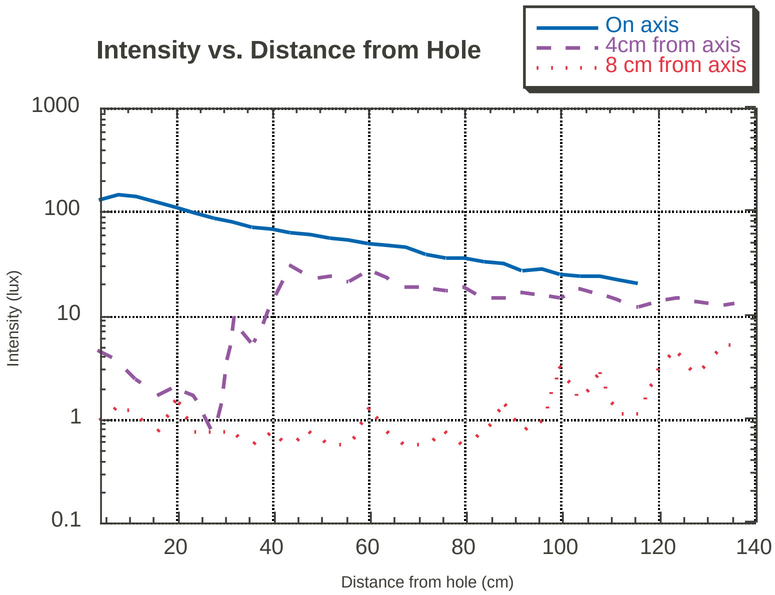

Figure 4: Light Intensity vs. Distance with No Lens

squares) for the on-axis data shown in figure 4 established that

$$\text{intensity} = \frac{9.16\text{x}10^5}{(r + 75.0)^2} \tag{1}$$

with a correlation coefficient of .988, where r is the distance from the hole in centimeters. Therefore the light intensity follows the inverse square law, as expected.

The two curves for off-axis data rise at first because only the light that passes through the hole is measured. Away from the axis of the lens and near the wall in which the hole was made, the wall blocks most of the light from the light bulb, and therefore the light intensity is very low. As the distance from the wall increases, the light spreads from the hole, increasing the intensity of the light off-axis. At distances greater than the location of the highest light intensity, the light intensity again approximately follows the inverse square law. For example, the best fit to the data taken at 4 cm off the axis and at distances between 45 cm and 135 from the hole is

$$\text{intensity} = \frac{9.36\text{x}10^5}{(r+144)^2} \tag{2}$$

with a correlation coefficient of .915.

Because these data are consistent with what is expected from basic properties of radiation, it confirms the validity of the experiment.

4.2 Lens

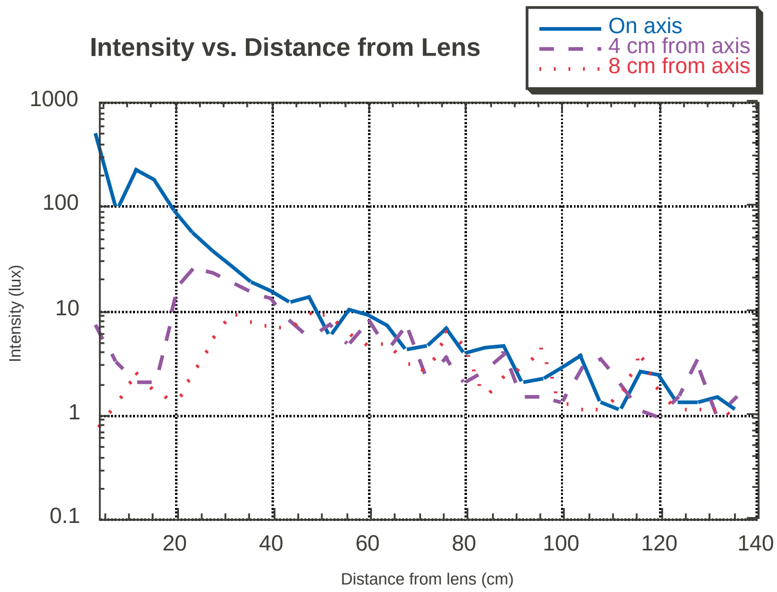

Figure 5: Light Intensity vs. Distance with a Lens

The light intensity data are considerably different when a lens is placed in the hole. First we note in figure 5 that all three graphs for light intensity versus distance have periodic ripples. The regularity of these ripples is suggestive of diffraction caused by constructive and destructive interference of light waves passing through different parts of the lens.

An obvious difference between the case of a lens and the case with no lens is that a lens induces a focal point on the axis of the lens where the intensity of light is greater than the intensity closer to the lens. In this case, we see that on axis the focal point is about 12 cm from the lens.

At large distances from the hole, the intensity of light passing through the lens as measured along the axis is much less than the intensity of light passing through the hole alone. For example, at a distance of 116 cm from the hole the intensity of light passing through the lens is 2.63 lux wheras the intensity of light passing only through the hole is 20.5 lux. However, the intensity of light passing through the lens also closely follows an inverse square law. For the data points taken at distances greater than the distance of the focal point from the lens, the best fit is

$$\text{intensity} = \frac{1.79\text{x}10^4}{(r - 6.00)^2} \tag{3}$$

with a correlation coefficient of .999. This is because the light concentrated at the focal point behaves approximately like a point source which radiates light in a cone.

4.3 Effect of Distance From the Lens Axis When the Lens Is Obstructed

When an opaque disk is used to block part of the lens, the light that hits the disk is absorbed while the light that hits an uncovered part of the lens is allowed to pass through. As seen in figure 6, this affects the distribution of the light intensity.

At each distance from the lens axis, the light intensity peaks at a small distance from the lens and then decreases rapidly. Along lines farther away from the lens axis, the light intensity peak is farther from the lens and lower than along the lens axis. In addition, the segment of high light intensity is longer off-axis than on-axis. Because the center

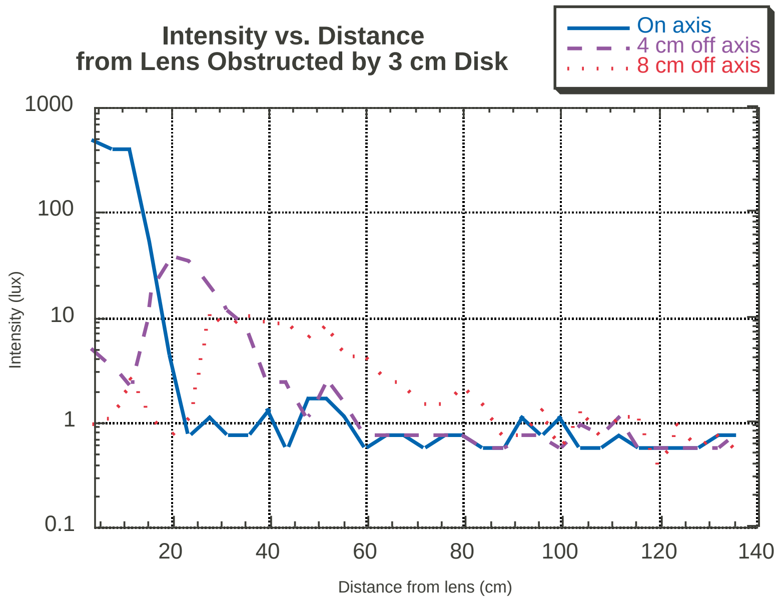

Figure 6: Light Intensity when the Lens Is Obstructed by a 3 cm Disk

of the lens is blocked, no light can pass through the center of the lens. This creates a conical region with vertex at the focal point and centered on the lens axis that contains points of low light intensity.

The light that passes through the outer rim of the lens is refracted to follow a straight path passing through the focal point. Therefore, the light intensity at the focal point is high but there is little light elsewhere along the axis of the lens. Blocking the center of the lens creates a shadow effect at most points along the lens axis.

As evident in figure 6, there is a similar effect along lines parallel to the axis of the lens. As the distance from the lens is increased, the light spreads out more, so the light intensity along a line parallel to the lens axis initially increases. Because the center of the lens is blocked, after a certain distance no more light rays meet the off-axis line, so the light intensity rapidly decreases. However, along off-axis lines the light intensity decreases more slowly than along the lens axis, because as the distance from the lens axis increases, the light rays spread out more. This creates longer segments of high intensity along lines parallel to lens axis.

4.4 Comparison of the Five Lens Conditions

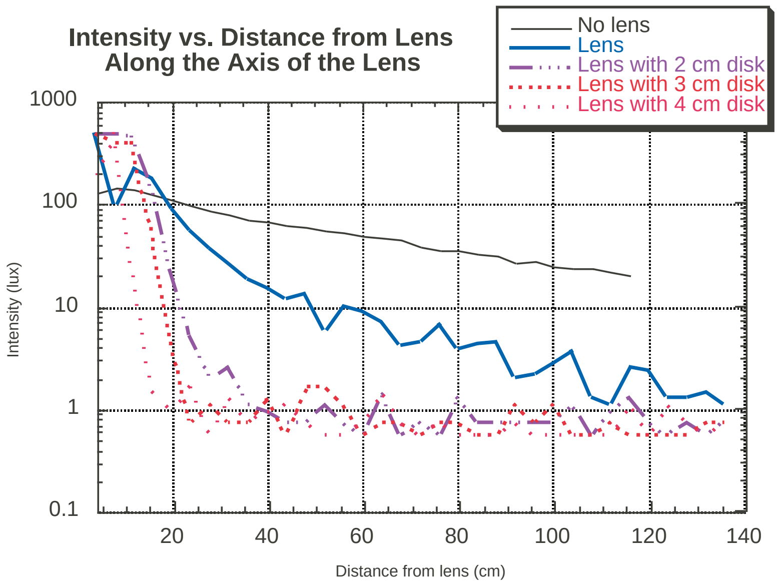

Figure 7: On-Axis Light Intensity for the Five Lens Conditions

Placing an obstructed or unobstructed lens in the hole has important effects on the light intensity distribution. Furthermore, the size of the disk used to obstruct the lens affects the speed at which the light intensity decreases as the distance from the lens increases.

In figures 7 and 8, we see that the locations of the on-axis and 8 cm off-axis focal points are preserved when any size of opaque disk blocks the center of the lens. However, in each graph the light intensity at the focal point is higher when the lens is obstructed than when it is not obstructed.

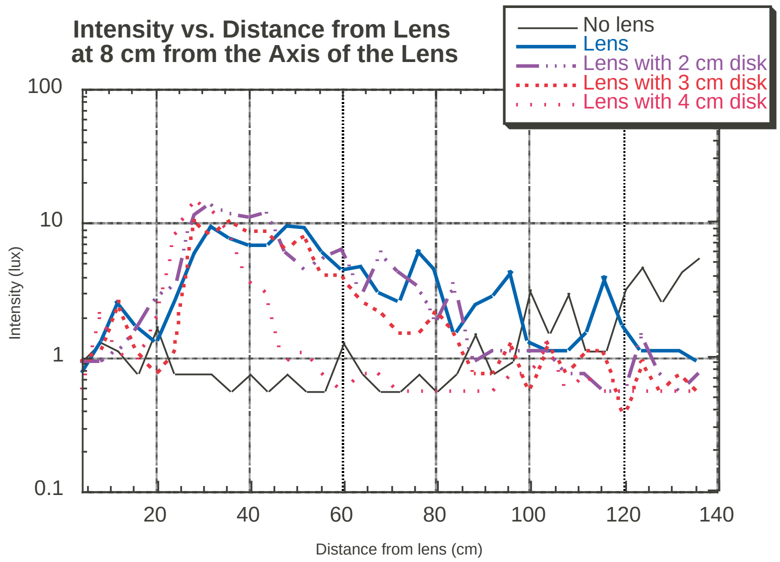

Figure 8: 8 cm Off-Axis Light Intensity for the Five Lens Conditions

However, when the lens is obstructed by a disk, the light intensity decreases more rapidly than when the lens is unobstructed. Furthermore, the larger the disk that obstructs the lens is, the faster the light intensity decreases as distance increases. This trend is particularly evident in figure 7, the graph of on-axis light intensity. Also, in figure 8, we see that when the lens is obstructed by the 4 cm disk, the light intensity falls much faster than when the lens is obstructed by the either of the two smaller disks.

5 Discussion

In all cases of lensed light, after the focal point the light intensity decayed much more quickly then when the light had not been lensed. Furthermore, the larger the disk that obstructed the lens, the faster the light intensity decayed after the focal point. This is logical: if a disk were only infinitesimally smaller than the lens, the light rays that passed through the lens would follow paths along the outside of a double cone. Neglecting scattering of light, the only point along the axis of the lens at which the light intensity would be nonzero would be at the vertex of the double cone. Note that the amount of light at this point would be small because in this case very little light passes through the lens.

Because a larger opaque disk covering the center of a given lens corresponds to a larger planet with a weaker gravitational field, the size of disk relates to the density of the gravitational body that is acting as a lens. Therefore, using the light intensity or gravity detected at varied points, one could find information about the density of the body that was deflecting the light or gravity waves. In particular, a faster decay of light intensity or gravity after the focal point would indicate a less dense deflector. Because light intensity and gravity intensity decay after the focal point, the light intensity or gravity at a given point could help find the distance from the observer to the lensing object.

The results of this experiment also apply to the calculation of the gravity acting on a point. Consider the simple universe shown in figure 9 which consists of a binary neutron star system, a large mass, and an observer.

If the quantity $R_1 + R_2$ is much larger than the distance between the two binary stars, the binary star system can be treated like a point mass. In this case, Newtonian physics

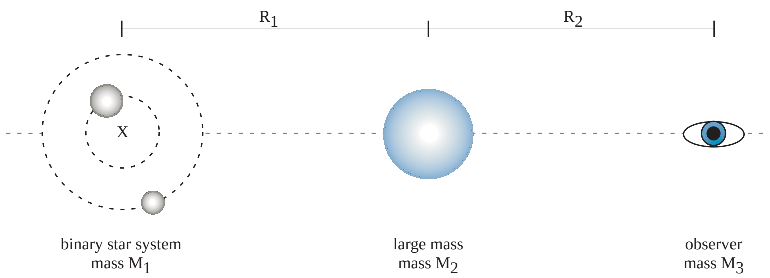

Figure 9: A Simple Universe

predicts that the gravity force acting on the observer is almost exactly

$$\frac{GM_1M_3}{(R_1+R_2)^2}+\frac{GM_2M_3}{(R_2)^2} \tag{4}$$

where G = 6.67x10^{-11}Nm^2/kg^2. Neglecting the effect of gravitational radiation, Einsteinian physics would predict a slightly different quantity. Including the effect of gravitational radiation, however, could make the gravity detected by the observer drastically different. Because a binary neutron star system also emits high frequency gravitational waves, the observer would also detect the oscillating gravity that was lensed by the large mass. This force would need to be added to the amount approximated by Newtonian physics. If the observer were at the focal point of the gravitational lens, the added amount could be very large. In general, based on the results of this experiment, the lensed gravity force would be a function of both the distance $R_1 + R_2$ and the density of the large mass.

In section 4.3 we noted that obstructing the lens created a conical region of very low light intensity. Gravitional lensing of either light or gravity would have a similar effect. Suppose that the large mass in the simple universe shown in figure 9 moved along a line perpendicular to the dotted gray line. If the observer were at the correct distance from the large mass, it would experience a flash of gravity as the large mass crossed the dotted gray line and created a gravitational focal point at the observer's location. However, if the observer were at a greater distance, the lensed gravity would be stronger when the large mass was not aligned with the binary star system and the observer. Therefore, as the large mass crossed the gray line, the observer would experience a flash of lensed gravity, then little lensed gravity as the large mass became aligned with the stars and the observer, and finally another flash of lensed gravity when the large mass moved away from the gray line.

Further experiments could refine the analogy between gravitational lensing and classical optics presented in this paper. In this experiment, a parabolic lens was used. Because the gravitational field of an object decays approximately proportionally with the inverse of the square of the distance from its surface, using a parabolic lens is not completely analogous to a gravitational field. Therefore, the experiment would be a better approximation of gravitational lensing of the thickness of the lens varied in a way that mimics the actual deflection caused by the inverse square law of gravity. Because gravitational lensing focuses light, such a lens would still be convex, but not parabolic.

In addition, by collecting data at smaller intervals, the diffraction effects that appeared in the graphs for this experiment could be studied. This would be interesting because it would help predict the intensity of high frequency gravitational waves affected by gravitational lensing. Furthermore, knowledge of light or gravity diffraction patterns could help predict the mass distribution of the deflector.

6 Conclusion

This experiment shows that the less dense an object is, the faster the intensity of light or gravity deflected by that object decays. This is useful in finding information about the density of the deflector and about the distance the deflector is from the observer. In addition, gravitational lensing causes the strength of the gravitational object to deviate from the strength that Newtonian physics predicts. Therefore the results of this experiment help understand how gravitational lensing behaves. Together with the advances in gravitational radiation detectors, an understanding of such lensing effects reveals the possibility of "gravitational radiation astronomy" – studying the universe in the light of gravity waves. These techniques may have applications in finding dark matter, estimating the size and age of the universe, and understanding many other aspects of the universe.

7 Acknowledgements

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